Application of the Planetary Geodesy Methods (the Geoid Theory) for the Reconstruction of the Earth's Interior Structure in the Western Antarctic

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Abstract

The algorithm for calculation of differential geoid heights and anomalous harmonic densities inside the Earth is represented in the paper. Expansion of external gravitational potential in series of spherical functions is used. The application of geopotential topography and anomalous harmonic densities for studing of structural features of regional deep sections is illustrated. The structure of the Scotia Arc Region with a technique named by us as the gravimetric tomography method is considered along typical latitudinal and longitudinal cross-sections, and also on maps of the differential geoid for different ranges of spherical harmonics.

1. Theoretical background

Density of the Earth's interior can be presented as a sum of normal and anomalous components:

$$\rho = \rho_i + \rho_{\grave{a}},$$

where normal density _ determines normal gravity potential of the Earth; disturbing potential is caused by anomalous density _.

The global gravity models describe the disturbing potential of the Earth, which depends on spherical coordinates of the point on its surface. It can be represented as Newton`s integral

$$V(r,\theta,\lambda) = G\iiint_V \frac{\rho}{l}\,dv,$$

where G – gravitational constant, l – distance from the investigated point to the surface of the Earth, v – elementary volume. So, there is a linear dependence between _ _ V, i. e. V = N_, where N is a linear operator.

Hence, the inverse gravitational problem can be formulated in the following way:

_=N^{-1}V,

the operator N^{-1} is unique if V is determined in the whole 3-dimensional space [Moritz, 1990]. The kernel of the operator N^{-1} consists of set of possible distributions of density _$_0$ inside closed contour S (within the Earth), which generate ground external potential.

The uniquely defned inverse Newton`s operator provides harmonic density _.

_ _=N_$^{-1}$V.

Thus, the solution could be found in the way of summation of definite harmonic density and the density of ground potential _$_0$.

$$\rho_a = \rho_0 + \rho_h$$

The densities of ground potential characterize features of spherical stratification inside the Earth. Local heterogeneity is described by anomalous harmonic density _$_h$.

2. The algorithm for calculation of the disturbing potential (or geoid height)

The geoid height above reference ellipsoid is found from the well-known formula

$$\zeta = R\sum_{n=2}^{z}\sum_{m=0}^{n}(c_{nm}\cos m\lambda + s_{nm}\sin m\lambda)P_{nm}(\cos\hat{A}),$$

where R≈6371 km – radius of the Earth, $\__{nm}$ and s_{nm} – normalized coefficients of external spherical harmonics of gravity potential, _ – longitude of the investigated point, _ – polar distance of the investigated point, $P_{nm}(\cos\_)$ – Legendre polynomial, z – number of harmonics to the 360th inclusive.

The formula taken from [Shabanova, 1962] was used for calculation of normalized spherical functions:

$$P_{nm}(\cos\hat{A}) = \cos\hat{A}\sqrt{\frac{4n^2-1}{n^2-k^2}}P_{n-1,m}(\cos\hat{A}) - \sqrt{\frac{2n-1}{2n-3}\cdot\frac{(n-1)^2-m^2}{n^2-m^2}}P_{n-2,m}(\cos\hat{A})$$

The necessary set of formules for calculation of any spherical function can be received on the base of following ones

$$\begin{aligned}
&P_{00}(\cos\hat{A}) = 1,\\
&P_{11}(\cos\hat{A}) = \sqrt{3}\sin\hat{A},\\
&P_{10}(\cos\hat{A}) = \sqrt{3}\cos\hat{A},\\
&P_{mm}(\cos\hat{A}) = \sin\hat{A}\sqrt{\frac{2m+1}{2m}}P_{m-1,m-1}(\cos\hat{A}), m > 1
\end{aligned}$$

3.The algorithm for calculation of anomalous harmomic densities

The solution of the inverse gravity problem is given by Moritz [1990] under condition that distribution of density is a continuous function which can be approximated uniformly with a system of polynomials.

The density as a function of spherical coordinates can be expansed in series of spherical harmonics

$$\rho(r,\theta,\lambda) = \sum_{n=0}^{\infty}\sum_{m=0}^{n}(a_{nm}\cos m\lambda + b_{nm}\sin m\lambda)P_{nm}(\cos B) = \sum_{n=0}^{\infty}\sum_{m=0}^{n}f_{nm}(r)Y_{nm};$$

The coefficients f_{nm} (=a_{nm} or b_{nm}) are arbitrary and can be represented as polinomials

$$f_{nm}(r) = \sum_{k=0}^{N}x_{nmk}r_k$$

Having excluded a general solution, which corresponds to the densities of ground potential, the expression for anomalous harmonic densities as a series of internal spherical harmonics is received

$$\rho_h(r,\theta,\lambda) = \sum\sum x_{nm}r^nY_{nm}(\theta,\lambda),$$

where $x_{nm} = \frac{(2n+1)(2n+3)}{4\pi GR^{2n+3}}V_{nm}$, V_{nm} – coefficients of external gravity potential,

$V_{nm}(= c_{nm}\cdot GMR^n$ or $s_{nm}\cdot GMR^n)$, M – mass of the Earth.

Thus, external spherical harmonics are used for determination of internal spherical harmonics. The internal spherical harmonics are used to receive distribution of positive and negative density inhomogeneities, which does not change external gravity potential as their total mass is equal to zero.

The final formula is

$$\rho_h = \sum_{n=2}^{z}\sum_{m=0}^{n}\frac{M(2n+1)(2n+3)}{4\pi R^{n+3}}\cdot r^n(c_{nm}\cos m\lambda + s_{nm}\sin m\lambda)P_{nm}(\cos B),$$

Corresponding number of harmonics z is taken to calculate the density $\__h$ on the depth (R-r).

4.Estimation of the depth of disturbing layer according to the number of harmonic

The obtained above density anomalies obviously are situated in different depths. However the mathematical methods used in physical geodesy though suppose but have no evident connection with the real geological structure of the Earth. Therefore a following question is remained: what layers of the Earth are responsible for disturbing gravitational potential.

In our method an assessment of the disturbing layer's depth is computed by a known harmonic function in physical geodesy in the case of the potential of the internal masses confined by a sphere

$$\frac{1}{r}=\sum_{n=o}^{\infty}\frac{\rho^{n}}{R^{n+1}}P_{n}(\cos\psi),$$

where r is the distance from the sphere surface down to the disturbing mass, ρ is the distance from the center of the sphere up to the disturbing mass, R is the radius of the sphere, P_n (cosΨ) is the Legendre polynomial of n^{th} degree, Ψ is the central angle between R and ρ.

Calculations was carries out from n_{min}=2. At the right part of the expression the normalizing coefficient $(2n+1)^{1/2}$ was considered. If Ψ=0, P_n (cosΨ) =1 for any n, _nd r = R-ρ. Under this condition for specified values of r, ρ and R corresponding n was found.

Obviously, for large n the series $\frac{\rho^{n}}{R^{n+1}}$ converge weakly. That is why restrictions are included into the procedure of calculation. Maximal degree n_{max} is defined under condition

$$r-\frac{R}{\sum_{n=0}^{n_{max}}\left(\frac{\rho}{R}\right)^{n}}\leq 0.1r,$$

where coefficient 0.1 means 10% error from r.

Relationship between harmonic degrees n and depths r of disturbing layers is shown on the bilogarithmic diagram (Fig. 1). Main boundaries of the lithosphere are shown in accordance with the Bullard's density model of the Earth.

The program allows to compute values of the disturbing gravity potential in terms of heights of the geoid, values of harmonic anomalies in units of g/cm3 and values of upper cover depths of disturbing layers of the Earth. Spherical coefficients of two geoid models OSU91A and EGM96 are used.

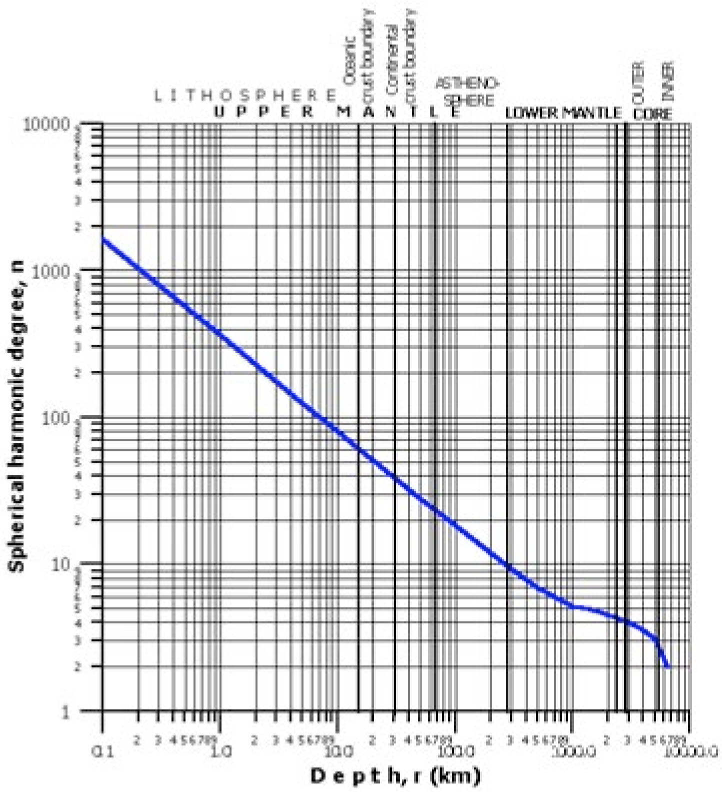

Figure 1. Relationship between harmonic degrees *n* and depths r of the disturbing layers of the Earth. Value n is the sum of harmonicas in a range from degree 2 up to n. Depth *r* corresponds to upper cover of the disturbing layer, which thickness is considered from the center of the Earth

5. Examples of density structure in the Scotia Arc region

Deep structure and geodynamics of the Scotia Arc and adjacent provinces within limits of 48S-66S and 80W-10W are submitted with the EGM96 gravity geoid model. The distribution of density inhomogeneities of the Earth is displayed along the Scotia Sea's central 58S latitudinal vertical cross-section (Fig. 2) and on 100 km and 1 km lateral levels with the spatial resolution of 30 km also (Fig. 3).

The images of differential anomalies (relatively of homogeneous deep layers) as three-dimensional surfaces show a detailed distribution of masses in the upper layers of the lithosphere, geometry and sizes of density inhomogeneities, their displacement in depth under influence of dynamic processes, and correlation of subsurface bodies with the bottom topography also.

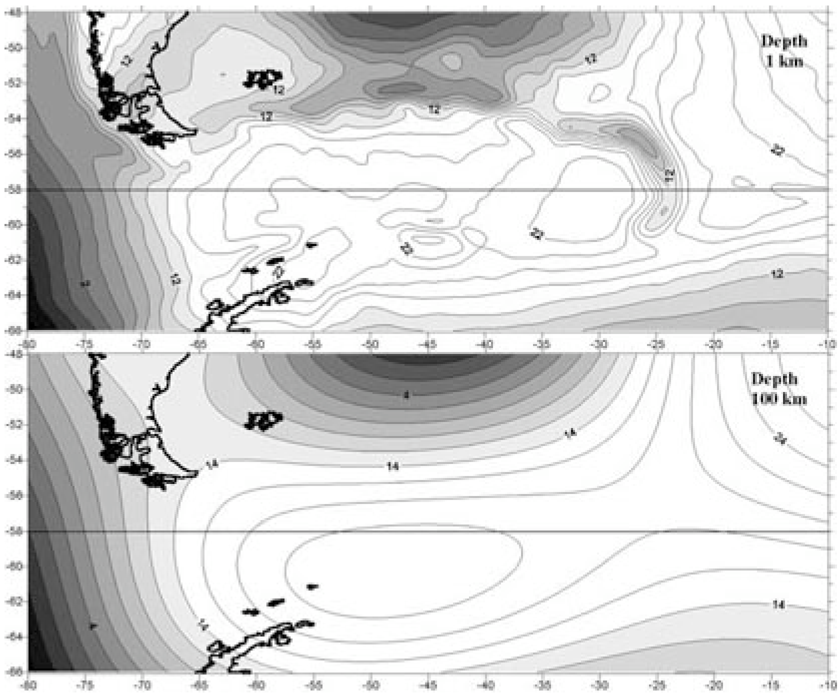

Figure 2. Maps of differential geoids topography corresponding disturbing layers with indicated depths of the upper cover. Lighter tint is more dense structure. The area of increased density is noted in depth of 100 km with epicenter 50S, 47W. It is a root part of the really Scotia body. On smaller depths (1 km) a structural differentiation of the Scotia Sea and correlation with the bottom relief are increased.

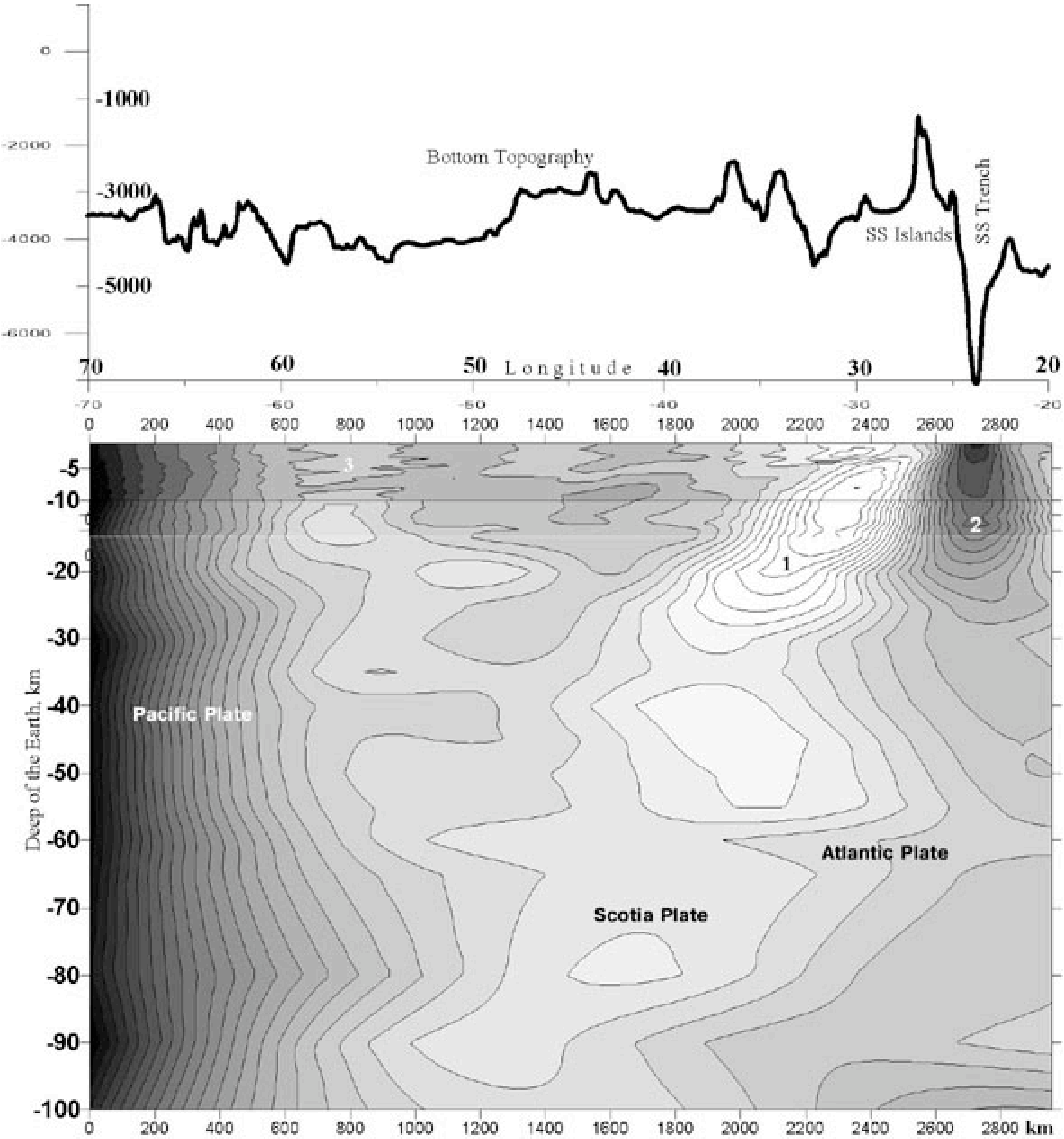

Figure 3. Vertical cross-section of the geopotential along 58S latitude.

The roots of the South Sandwich Island ridge (1) are sloped to the Scotia Sea side and are immersed into depths 25-30 km. The SS Trench (2) is traced up to depth of 20 km. A fragment (3) of a joint area of the Shackleton F.Z. and the Western Scotia Ridge is observed at 57W.

References

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2. Shabanova A I (1962) Calculation on ECVM «Ural-1» of characteristics of gravity field applying gravity anomalies in spherical functions (in Russian) J CNIIGAiK 145: 77-81.